P' AND THE EARTH'S CORE.

B. Gutenberg and C. F. Richter.

(Received 1938 March 10)

The velocity of longitudinal seismic waves in the Earth increases with depth down to the discontinuity separating mantle and core, at a depth of about 2900 km., where it drops from 13·7 to about 8 km./sec. Within the core, the velocity has usually been calculated as increasing gradually to about 11·5 km./sec. at the centre.

This velocity distribution corresponds to a well-known focusing effect for longitudinal waves which have passed through both mantle and core, designated by the symbol P_cP_cP, PKP or P'. The focal curve, or caustic, occurs at about 142° from the epicentre. The interval between this focus and the last arrival of direct longitudinal waves through the mantle (P), at about 105°, should be a shadow zone. The P waves, however, are known to continue into this interval by diffraction around the core.* They appear as waves of long period (usually over 10 seconds), with small amplitudes, which decrease still further with increasing distance.

Apparently the P' waves also extend into the shadow zone; numerous observations fall on a curve or group of curves which extend backward from the focus at 142°, at least as far as 110°. This phenomenon was first pointed out by Rudolph and Szirtes † in a paper based on the observations of the South American earthquake of 1906 January 31. They interpreted these observations as lying on a curve branching from that for PP. This interpretation was adopted by Angenheister,‡ who added many new observations, and suggested displacing the point of branching from PP to a somewhat shorter distance.

This matter was discussed by Wiechert in a paper presented at the *Naturforscherversammlung* at Leipzig, 1922, but not published. He proposed to explain the observations as due to diffracted P' waves analogous to the diffracted P; this explanation has been repeated by several other authors, and has been generally accepted in the literature. However, the process of such a diffraction is rather obscure; no one has yet given an exact quantitative theory to cover the case. Moreover, these observed waves are strikingly dissimilar in character from the observed diffracted P waves; their periods are very short (one or two seconds), and the ampli-

* B. Gutenberg, "Ueber Erdbebenwellen VII A," *Göttinger Nachrichten*, 166, 1914.

† E. Rudolph and S. Szirtes, "Ueber eine neue Laufzeitkurve (vorläufige Mitteilung)," *Physikalische Zeitschrift*, 15, 737–739, 1914.

‡ G. Angenheister, "Beobachtungen an pazifischen Beben," *Göttinger Nachrichten*, 113–146, 1921.

Downloaded from https://academic.oup.com/gsmnras/article-abstract/4/5/363/653490 by California Institute of Technology user on 11 November 2019

tudes in the vertical component are often very large, especially at certain critical distances.

The difficulties have lately been emphasized by Miss Lehmann,* who suggests that the phenomena may be explained by the existence of a discontinuity within the core. She calculates travel times for a hypothetical model with constant velocities of 10 km./sec. in the mantle, 8·0 km./sec. in the outer part of the core and 8·6 km./sec. in the inner core, the radius of the inner core being 0·2205 of the radius of the Earth, or 1400 km. The results agree qualitatively with the characteristics of the observed curves. However, no attempt is made to determine quantitatively the velocities required in the actual earth to represent the observed times and amplitudes. She points out that a first-order discontinuity within the core is not necessary, and that a rapid but continuous increase in velocity at about the level indicated is consistent with the data.

For the purpose of the following discussion we need a notation for the branch or branches of the P' curve observed between 110° and 142°, previously referred to as the "diffracted" P'. We here adopt the provisional notation P''. On first examination the observations of P' then fall on three main branches, P_1', P_2' and P'', with some indication of other branches. The present hypothesis requires that none of the three main branches is simple, so that a further revision of notation will be called for when the matter is satisfactorily cleared up. It is suggested that the decision as to such a notation be left to the forthcoming meeting of the International Seismological Association.

We accept Miss Lehmann's hypothesis that the observations of P'' are not due to diffraction, but to waves refracted in a normal manner through the core. However, the velocity distribution required to account for these observations differs greatly from the hypothetical model used in her paper. The most direct approach to this velocity distribution is by way of the method used by Wadati and Masuda.† In our second paper on seismic waves ‡ we have extended this method, and used it to determine velocities within the core. Travel times between points on the surface of the core were derived from P_1', P_2' and other phases, but not from P''.

In selecting observations of P'' for the application of this method, it must be noted that the observed points do not fall on a single well-defined curve. There is evidence that more than one branch is involved, and apparently the general causes of multiplicity are also at work here. Times for a number of the suggested curves were tabulated in our first paper on seismic waves.§ In this range of times and distances there are a number of

* I. Lehmann, "P'," *Publications du Bureau central séismologique international*, Série A : Travaux scientifiques, fasc. 14, 87–115, 1936.

† K. Wadati and K. Masuda, "On the Travel Time of Earthquake Waves, Part VI," *Geophysical Magazine*, Tokyo, **8**, 187–194, 1934.

‡ B. Gutenberg and C. F. Richter, "On Seismic Waves (second paper)," *Gerlands Beiträge zur Geophysik*, **45**, 280–360, 1935.

§ B. Gutenberg and C. F. Richter, "On Seismic Waves (first paper)," *Gerlands Beiträge zur Geophysik*, **43**, 56–133, 1934.

Downloaded from https://academic.oup.com/gsmnras/article-abstract/4/5/363/653490 by California Institute of Technology user on 11 November 2019

observed waves, which from their longer periods and different apparent velocities cannot be taken as part of the P'' group. Such are the observations represented by the curves lettered H, I, J, K, N in the paper just referred to. The same is probably true of parts of the earlier curves there tabulated for the "diffracted P'," especially No. 2 and No. 3. The earliest points properly acceptable as P'' are at about 110° with travel time about $18^m\ 30^s$. Selecting a few points, and estimating the apparent velocity associated with each, we subtract from the distance and travel time of P'' the distance and travel time of the corresponding *PcP* having the same apparent velocity; the result is a distance and a travel time between two points on the surface of the core. This curve is to be combined with that found from P_1', P_2', etc., in the second paper on seismic waves (*loc. cit.*, Tables XXIII and XXIV).

The new curve appears to branch from the old one, in the direction of decreasing distance, at about 120°. This suggests a curve of the type considered by Miss Lehmann, which is produced by a rapid decrease in velocity with depth, not necessarily involving a discontinuity. Such a curve has a loop, with a reversed segment. It should be noted that the travel-time curves here referred to are for points on the surface of the core, which are somewhat distantly related to the ordinary curves for points on the surface of the Earth, such as are drawn by Miss Lehmann.

As pointed out by Slichter,* the method of Herglotz, Bateman and Wiechert can be applied to determine the velocity at depth from such a travel-time curve, even when there is a loop, provided that the integrals are taken as negative over the reversed segment. A first application of this method showed that the entire curve, including the branch derived from P'', can be accounted for by a single consistent velocity distribution, which can be made smooth and continuous by very slight adjustments, not exceeding a few seconds, in the assumed observational data. Small differences within the limits of observational error are capable of producing large differences in the calculated results. However, the data definitely require a rapid increase in velocity from slightly more than 10 km./sec. at radius 1500 km. (depth 4866 km.) to almost 11·5 km./sec. at radius 1200 km., the velocity being nearly constant in the central part of the core.

The first rough calculations have been improved by the method of trial and error, the adjustments in the assumptions always being very small. Additional data on travel times and apparent velocities have been taken from a number of recent shocks, particularly from large deep-focus shocks south of Borneo, recorded on the Benioff short-period vertical instruments at Berkeley, Tucson,† and Pasadena and its auxiliary stations. These seismograms show that the amplitudes of P'' are extremely large in a limited range of distance about 122°. A similar maximum for pP'' occurs at slightly larger distances, as is to be expected. Thus, for the shock of 1937 August 11

* L. B. Slichter, "The Theory of the Interpretation of Seismic Travel-time Curves in Horizontal Structures," *Physics*, **3**, 273–295, 1932.

† We are indebted to Professor Perry Byerly of the University of California, and to the United States Coast and Geodetic Survey, for the loan of the Berkeley and Tucson records, respectively.

Downloaded from https://academic.oup.com/gsmnras/article-abstract/4/5/363/653490 by California Institute of Technology user on 11 November 2019

(depth about 600 km.) the seismogram at Pasadena, distant about 123°, shows a large P'' and a smaller but evident pP'' (Plate 3); Tinemaha, distant about 121½°, shows a large P'', while pP'' is apparently wanting; Tucson, at about 129°, shows a large P'' and a very large pP''. Large amplitudes also occur about 111° (compare fig. 6 of our first paper on seismic waves), and have been found by Miss Lehmann (*loc. cit.*) near 135°. It is possible that these cases of large amplitudes differ in different shocks and with different paths.

In making further adjustments we have considered these large amplitudes, in so far as we have tried to make them correspond to large curvature in the travel-time curve (rapid change in apparent velocity). A nearly straight travel-time curve (nearly constant apparent velocity) should correspond to small amplitudes. Now, a large change in the apparent velocity corresponds to a rapid rate of increase in the actual velocity at the deepest point reached by the seismic ray; if the effect is very large, resulting in large amplitudes, it requires that the rate of increase in velocity with depth shall approach the critical rate which is sufficient to bring successive rays to the same point. Such a focusing effect does not occur in case of a discontinuous change in velocity, so that both observed amplitudes and travel times suggest a rapid but continuous increase in velocity in a particular narrow range of level within the core, rather than a discontinuity. Moreover, no reflected waves have been found such as would correspond to a discontinuity.

If the rate of increase in velocity remains close to the critical value throughout a thin shell in the core, variations in this rate within the shell may result in large amplitudes at several points on the surface, distributed over a considerable range of epicentral distance. The locations of these quasi-focal points will be extremely sensitive to the variations in conditions within the critical shell. Consequently any cause which slightly changes the direction of path through this shell will materially affect this phenomenon. In particular, the ellipticity of the Earth may result in widely different effects in different azimuths.

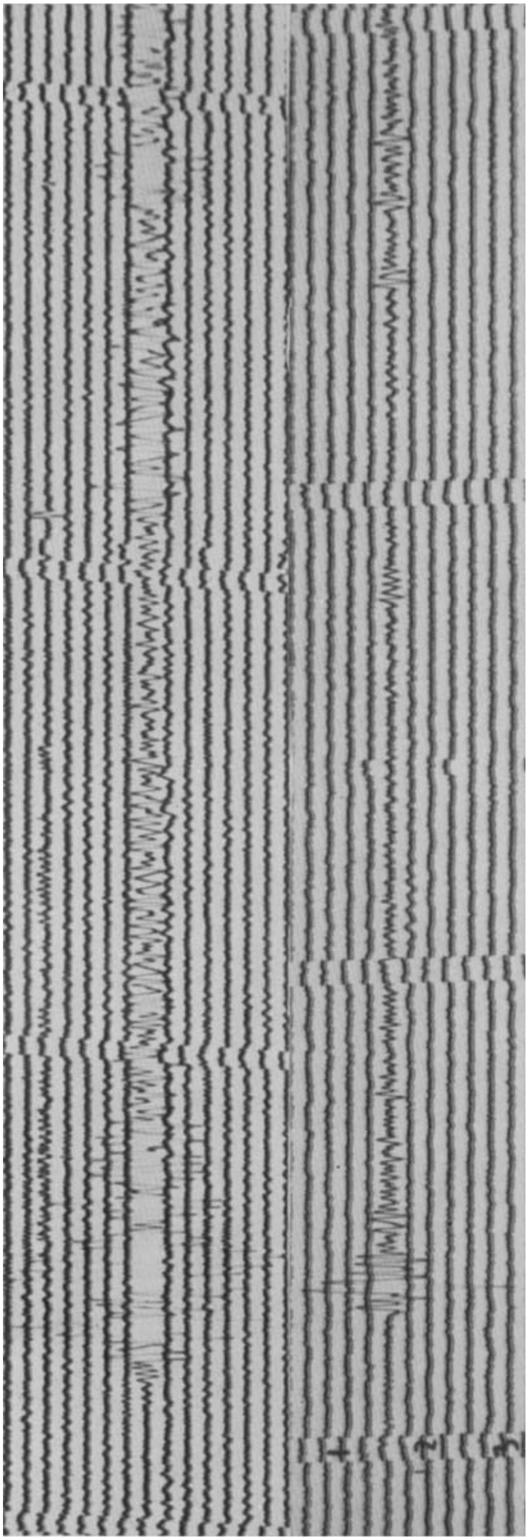
We have endeavoured to adjust our hypothesis to represent the large amplitudes by such a critical condition; however, there is a further consideration which closely limits the range of possible assumptions, namely, any two branches of the travel-time curve which meet at a cusp must have a common tangent (equal apparent velocity) at that cusp; and all parts of the travel-time curve must be convex toward the axis of distances.

It is found that the new assumptions permit an improved representation of the observed times at the focus of P' (about 142°). Formerly it was not possible to represent the focal point correctly without too steep a P_1' curve; the observed times were decidedly early compared with the rest of the P' observations, and consequently earlier than the calculated times.

Some adjustments were made in order to get an acceptable calculated distance (130°) for the focal point of *PKS*.

Table I gives our adopted travel times between points on the surface of the core with the corresponding reciprocals of the apparent velocity

Downloaded from https://academic.oup.com/gsmnras/article-abstract/4/5/363/653490 by California Institute of Technology user on 11 November 2019



Seismograms at Pasadena, Benioff short-period vertical. (Above) 1937 August 11, 0^h. Epicentre south of Borneo, depth of focus about 600 km., distance about 123°. (Below) 1936 May 8, 9^h. Epicentre 6° S., 113° E.; depth 620 km., distance about 125°.

Messrs. B. Gutenberg and C. F. Richter, P′ and the Earth's Core.

Downloaded from https://academic.oup.com/gsmnras/article-abstract/4/5/363/653490 by California Institute of Technology user on 11 November 2019

TABLE I

Adopted Travel Times t between Points on the Surface of the Core, and Reciprocals $1/\bar{v}$ of the Apparent Velocity

Θ Degree	t min. sec.	$1/\bar{v}$ sec./km.	Θ Degree	t min. sec.	$1/\bar{v}$ sec./km.	Θ Degree	t min. sec.	$1/\bar{v}$ sec./km.
2	0 15	0·125	72	7 42	0·079	98	9 48	0·032
4	0 30	125	74	7 51	77	96	9 44	315
6	0 45	125	76	8 00	75	94	9 40	315
8	1 00	125	78	8 09	73	92	9 37	315
10	1 15	124	80	8 18	71	90	9 33	315
12	1 30	124	82	8 26	70	92	9 37	315
14	1 45	124	84	8 35	68	94	9 41	315
16	2 00	124	86	8 43	66	96	9 44	315
18	2 15	123	88	8 51	65	98	9 48	315
20	2 30	122	90	8 58	63	100	9 52	315
22	2 44	121	92	9 06	61	102	9 56	31
24	2 59	119	94	9 13	59	104	9 59	31
26	3 13	117	96	9 20	57	106	10 03	31
28	3 27	115	98	9 27	555	108	10 07	31
30	3 41	113	100	9 33	54	110	10 11	305
32	3 54	111	102	9 40	53	112	10 14	305
34	4 07	109	104	9 46	52	114	10 18	305
36	4 20	107	106	9 52	51	116	10 21	305
38	4 33	105	108	9 59	50	118	10 25	305
40	4 46	103	110	10 05	49	120	10 29	30
42	4 58	101	112	10 10	48	122	10 32	30
44	5 10	100	114	10 16	47	124	10 36	30
46	5 22	098	116	10 22	46	126	10 40	295
48	5 34	096	118	10 27	45	128	10 43	295
50	5 45	095	120	10 33	44	130	10 47	29
52	5 56	093	118	10 28	40	132	10 50	285
54	6 08	092	116	10 24	36	134	10 54	28
56	6 19	091	114	10 20	345	136	10 57	27
58	6 30	090	112	10 15	34	138	11 00	26
60	6 41	089	110	10 11	335	140	11 03	25
62	6 51	088	108	10 07	335	150	11 16	16
64	7 02	086	106	10 03	33	160	11 24	10
66	7 12	084	104	9 59	33	170	11 28	04
68	7 22	083	102	9 56	325	180	11 29	00
70	7 32	081	100	9 52	32			

Downloaded from https://academic.oup.com/gsmnras/article-abstract/4/5/363/653490 by California Institute of Technology user on 11 November 2019

at the surface of the core. The length of 1° on the surface of the core has been taken as 60·21 km.; this corresponds to a depth for the core of 2920 km., or a radius of 3446 km. The values have been adjusted to make

$$t=\int_0^{\Theta}\frac{1}{\bar{v}}\,60{\cdot}21\,d\Theta.$$

These travel times may be compared with those given in our previous publications, and also with those more recently given by Jeffreys.* The differences are within the limits of error. For the first 35° the times are up to 4 seconds longer than those given by Jeffreys; for larger distances they are shorter, the difference not exceeding 7 seconds. With a few exceptions, these new times are closer to Jeffreys's times than our older ones. This is very satisfactory because, as already indicated, the new times have been worked out primarily to provide an explanation of P'', and adjusted principally to fit the observed times of that phase. The intersection of the two branches of the new curve near 115° is so flat that the differences against Jeffreys's times show no evident irregularity; the departure from a single continuous curve is too slight to be established solely by observations of waves with central angle through the core near 115°.

The travel-time curve for the core thus has a loop; the three branches result in two cusps at the indicated points (120° and 90°) where the times reverse with decreasing angle of incidence. The Wiechert-Herglotz-Bateman method has been applied to this curve; the depth reached by each ray, and the velocity at that depth, are given in this case by

$$\log r_s=3{\cdot}5373-0{\cdot}0024127\int_0^{\Theta}\cosh^{-1}q\,d\Theta,\quad v_s=\frac{r_s\bar{v}}{3446},$$

where r_s is the minimum radius vector, v_s the corresponding velocity, $\bar{v}$ the apparent velocity at the surface of the core, and q is the ratio of apparent velocities as ordinarily used. Θ is the distance over the core in degrees.

The velocity distribution calculated is given in fig. 2. The abscissa is given in depth below the surface of the Earth at the top of the figure, and in radial distance from the centre at the bottom. As has been mentioned on previous occasions, large changes in the velocities are consistent with the data, and are well within the limits of error. The value near the surface of the core will depend principally on the observational data used for *SKS*. If the times used are later than the earliest *SKS* readings, the velocity just below the surface of the core will be nearer 7·5 than 8 km./sec. as given here.

Combining the travel times in Table I with times for waves in the mantle, we can derive the travel times for core waves reaching the surface. We have used the data for *PcP*, *ScS* and *ScP* given in our second paper on seismic waves. In connecting the segment in the core with those in the mantle,

* Harold Jeffreys, "On Travel Times in Seismology," *Publications du Bureau Central Séismologique International*, Série A, fasc. 14, 78, 1936.

Downloaded from https://academic.oup.com/gsmnras/article-abstract/4/5/363/653490 by California Institute of Technology user on 11 November 2019

we have taken the velocities for P and S just outside the core as 13·7 and 7·3 km./sec.

Tables II, III and IV give the calculated travel times for P', *SKS* and *SKP*.

The travel-time curve for P' has three cusps. The first segment, from 183° (177°) to $142\frac{1}{2}$°, is the old P_2', without significant modification. $142\frac{1}{2}$° is the focal point, with a time now brought into better agreement with the observations. No change is required in the former explanation

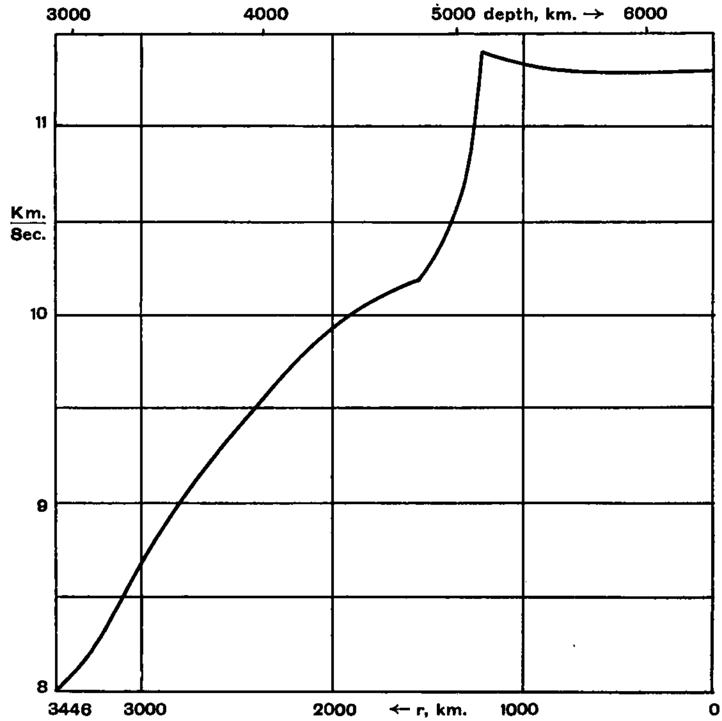


FIG. 2.—*Velocity of waves within the core.*

of this focusing effect. The second segment, from $142\frac{1}{2}$° to 152°, represents the stronger points of P_1'. However, the apparent velocity is less than that previously supposed. The observations suggest an extension to greater distances ; this may be due to diffraction. The third segment extends from 152° back to 110°. At the larger distances it is very close to the second segment ; between 142° and 110° it represents the earliest observations of P''. The fourth segment extends from 110° to 180°. At the shorter distances it is close to the third segment, which it precedes by a few seconds at most. Beyond 142° it accounts for a few well-established early observations of P_1', particularly for those near 150°, $19^m\ 44^s$, and 162°, $19^m\ 55^s$, which determine the No. 1 P' curve of our first paper on seismic waves.

The *SKP* curve is analogous to that for P' ; it also has three cusps dividing it into four segments. The principal focal point is at 130°, $22^m\ 31^s$

Downloaded from https://academic.oup.com/gsmnras/article-abstract/4/5/363/653490 by California Institute of Technology user on 11 November 2019

Most of the strong observed points are near this focus; however, there are enough to indicate the reality of the first and second segments. There is a considerable scatter of observations between 130° and 145°; most of these are late, but there are a few early points which may belong to late branches

TABLE II

Calculated Travel Times of P′

Δ	t	Δ	t	Δ	t
°	m s	°	m s	°	m s
183	22 18	150	19 52	125	18 55
180	22 05	145	19 38	130	19 04
170	21 21	140	19 25	135	19 14
160	20 38	135	19 15	140	19 23
155	20 16	130	19 06	145	19 32
150	19 55	125	18 56	150	19 41
145	19 37	120	18 47	155	19 49
142½	19 29	115	18 37	160	19 55
145	19 36	110	18 28	165	20 00
150	19 51	115	18 37	170	20 04
152	19 57	120	18 46	180	20 06

TABLE III

Calculated Travel Times of SKP

Δ	t	Δ	t	Δ	t
°	m s	°	m s	°	m s
145	23 34	125	22 30	135	22 48
140	23 12	120	22 20	140	22 57
135	22 51	115	22 11	145	23 06
130	22 31	110	22 02	150	23 14
135	22 49	105	21 53	155	23 22
140	23 04	110	22 02	160	23 29
144	23 15	115	22 11	165	23 33
140	23 05	120	22 20	170	23 36
135	22 53	125	22 29	175	23 37
130	22 41	130	22 30	180	23 37

of *PP*. The data used in our previous papers would not suggest drawing the third and fourth branches. However, excellent observations of these branches of *SKP*, corresponding to *P″*, are available for the South Atlantic earthquake of 1935 May 14. The epicentre was about 59° S., 26½° W.; the depth of the focus was between 100 and 200 km.; for the origin

Downloaded from https://academic.oup.com/gsmnras/article-abstract/4/5/363/653490 by California Institute of Technology user on 11 November 2019

time we find $23^h\ 23^m\ 10^s$. The following observations, referable to *SKP*, have been reported by the stations :—

Station	Δ	Travel Time
	°	m s
Philadelphia	105½	21 46
Strasbourg	111½	21 59
Praha	114	22 02
Pasadena	119	22 03
Riverside	119	21 59
Tinemaha	121½	22 08
Berkeley	124½	22 03
Honolulu	130	22 10
Manila	130	22 08
Seattle	132½	22 17
Hong-Kong	134	22 22

The observations begin exactly at the distance where they should be expected from the calculated times of Table III. The first three travel times are 8, 6 and 8 seconds earlier than those of the table; the rest are from 15 to 25 seconds earlier. The correction for depth of focus should be about 25 seconds. Some of these observations may refer to *PKS*, which should be later.

Table IV

Calculated Travel Times of SKS

Δ	*t*	Δ	*t*	Δ	*t*
°	m s	°	m s	°	m s
80	22 32	130	26 17	125	26 04
85	23 02	125	26 06	130	26 13
90	23 30	120	25 56	135	26 22
95	23 56	115	25 47	140	26 32
100	24 20	110	25 37	145	26 41
105	24 42	105	25 27	150	26 49
110	25 02	100	25 18	155	26 57
115	25 23	105	25 27	160	27 02
120	25 43	110	25 36	165	27 05
125	26 01	115	25 45	170	27 06
130	26 16	120	25 55	180	27 07
135	26 30				

For *SKS* the times for short distances have not been recalculated, as there are no dependable observations of *SKS* nearer than 80°. The curve has two cusps, dividing it into three segments. The earliest observations correspond approximately to the earliest parts of the calculated curve (beginning of the first segment, end of the third segment). The remaining parts of the calculated curve pass through a range where there are very many observations of all kinds, due to various phases and probably to multiplicity.

Downloaded from https://academic.oup.com/gsmnras/article-abstract/4/5/363/653490 by California Institute of Technology user on 11 November 2019

It is evident that the velocity distribution now assumed introduces two additional cusps into the travel-time curve for every wave passing through the core. A further instance of this is *PKKP*. The calculated curve begins at 100°, with a travel time of $30^m\ 26^s$; the first cusp is at the principal focal point ($123\frac{1}{2}°$, $28^m\ 42^s$), which corresponds with the observations much better than the value previously calculated without the modified velocity distribution. Two new cusps are at 88°, $30^m\ 30^s$, and $158\frac{1}{2}°$, $28^m\ 03^s$. From the last cusp the curve turns back to 0°, $27^m\ 07^s$.

All calculated times given in this paper are subject to a number of small uncertainties, which may easily accumulate to as much as ten seconds, and even somewhat more. As has been mentioned in previous papers, the travel times and velocity distribution for *S* in the mantle are in course of revision. When these are better determined, a recalculation for the core waves will be undertaken.

Summary

Waves formerly identified as diffracted *P′* can be accounted for more satisfactorily as ordinary refracted waves, provided that the velocity distribution within the core is assumed to be as follows: velocity increasing from about 8 km./sec. at the inside surface of the core to about 10·2 km./sec. about 1950 km. deeper, where there is a rapid but probably continuous increase with depth, the velocity reaching 11·4 km./sec. within 300 km., then decreasing slowly to 11·3 km./sec. at the centre.

This hypothesis is better in accordance with the occasional large amplitudes. The modified velocity distribution requires two additional reversals in the travel-time curve, which accounts for peculiarities in the observations of *P′* near the focal distance (which remains unchanged at 142°). It also accounts for similar observations of *SKP* and *P′P′*, and is consistent with the observations of other waves passing through the core.

Balch Graduate School of the Geological Sciences,
California Institute of Technology, Pasadena, Cal.,
Contribution No. 249*:*

1938 *February* 9.

Downloaded from https://academic.oup.com/gsmnras/article-abstract/4/5/363/653490 by California Institute of Technology user on 11 November 2019